2023 年第三十八屆成大數理比賽 –物理
38th NCKU Science and Mathematics Competition 2023 – Physics

1. Consider a conducting loop carrying current I under the influence of a magnetic field $\mathbf{B}$, as shown in Figure 1a. The loop experiences a torque given by

$$\vec{\tau} = I\,\mathbf{A} \times \mathbf{B},$$

where $\mathbf{A}$ is the vector of the loop area indicated in the sketch. Which of the following is correct?

A) The equation is valid for loops of arbitrary shapes (*not only rectangular*)

B) The torque in the sketched case rotates the loop counterclockwise (looking toward the x direction)

C) The two vectors $\mathbf{A}$ and $\mathbf{B}$ are related to each other via $\mathbf{B} = \nabla \times \mathbf{A}$

D) The vector $\mathbf{A}$ is also known as the vector potential

E) For the torque to be nonzero, $\mathbf{B}$ must be time-varying

考慮一帶電流 I、於磁場 $\mathbf{B}$ 作用下的線圈,如Figure 1a 所示。線圈感受到的力矩為

$$\vec{\tau} = I\,\mathbf{A} \times \mathbf{B}$$

其中 $\mathbf{A}$ 是線圈面積向量,如圖所示。下列敘述何者正確?
A) 該方程式可用在任意形狀的線圈(不侷限於方形)
B) 朝 x 軸看過去的話,力矩將線圈往逆時針方向轉動
C) 兩向量 $\mathbf{A}$ 與 $\mathbf{B}$ 之間的關係是 $\mathbf{B} = \nabla \times \mathbf{A}$
D) $\mathbf{A}$ 亦稱為向量勢(vector potential)
E) 要讓力矩不為零,則$\mathbf{B}$必須隨時間改變

2. About magnetic force $\mathbf{F}_m$ acting on a particle carrying charge q and moving at velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$, which of the following is **incorrect**?

A) The force can be written as $\mathbf{F}_m = q\mathbf{v} \times \mathbf{B}$

B) If $\mathbf{B} = (0, 0, B)$ with $B > 0$ and $\mathbf{v} = (v_x, v_y, 0)$, then the force is pointing to the right relative to $\mathbf{v}$ for positive charges, and left for negative charges

C) Because $\mathbf{F}_m \perp \mathbf{v}$, the force cannot change the kinetic energy of the charged particle

D) The Biot-Savart law describes such a magnetic force

E) The underlying mechanism of the cyclotron motion of electrons is such a magnetic force

關於磁場$\mathbf{B}$產生之磁力$\mathbf{F}$m,作用於帶電 q、以速度$\mathbf{v}$ 運行的粒子,下列敘述何者不正確?

A) 磁力可表示為$\mathbf{F}$m $= q\mathbf{v} \times \mathbf{B}$
B) 若$\mathbf{B} = (0, 0, B)$且$B > 0$、$\mathbf{v} = (v_x, v_y, 0)$,則相對 於運動速度的方向,磁力若是作用在帶正電的粒 子是朝右,作用在帶負電的粒子則是朝左
C) 因為$\mathbf{F}_m \perp \mathbf{v}$,磁力不能改變粒子的動能
D) Biot-Savart定律即描述此磁力
E) 電子的迴旋運動,背後的機制便是此磁力

3. Consider the plots shown in Figure 1b for a conducting sphere of radius R with charge $Q > 0$ uniformly distributed on its surface. If the magnitude of the electric field is $E(r)$ and the electric potential is $V(r)$, where r is the distance to the center of the sphere, which of the following is correct?

A) The conducting sphere should be neutral charge
B) $f = E$ and $g = V$
C) $f_0 = \sigma/\epsilon_0$ with $\sigma = Q/4\pi R^2$
D) The electrostatic energy stored is $\frac{1}{2}Qf_0$
E) If the conducting sphere is replaced with a dielectric sphere, the curves for V and E remain the same

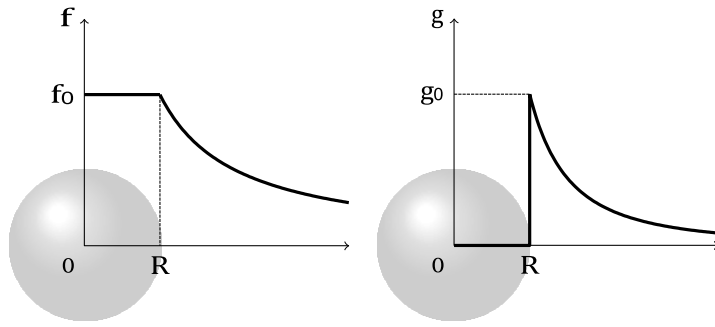半徑 R 的導體球,電荷 Q 均勻分布於其表面,其電 場強度 E 與電位 V 隨徑向距離(至球心的距離)變 化的情形如Figure 1b所示。下列何者正確?

A) 導體球應為電中性
B) $f = E$ 且 $g = V$
C) $f_o = \sigma/\epsilon_o$ 且 $\sigma = Q/4\pi R^2$
D) 所儲存的靜電能為$\frac{1}{2}Qf_0$
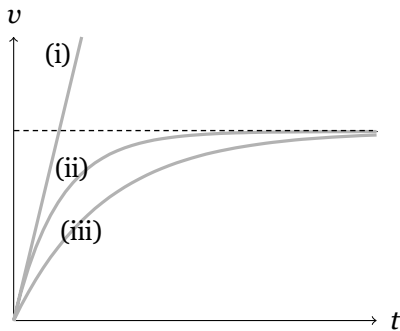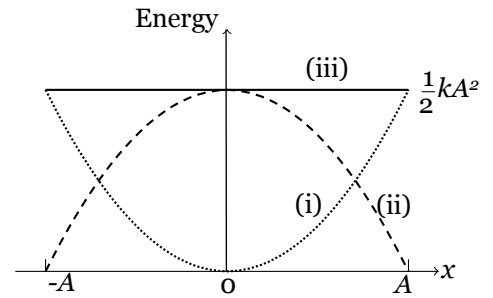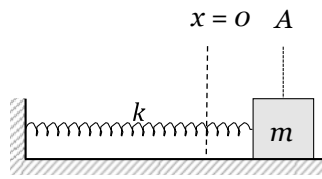E) 若將導體球置換為介電質球體,則V 與 E 的曲線 不變
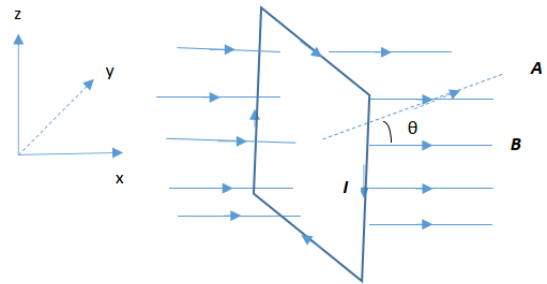
(a) Question 1

Figure 1

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(b) Question 3

(c) Question 4

(d) Question 5

Figure 1

4. The curves (i)–(iii) shown in Figure 1c describe the speed v of a falling body (not too far from the surface of the earth) as a function of time t. The slope of the straight line (i) is equal to $g = 9.8$ m/s^2. Which of the following is correct?

   A) None of the curves (i)–(iii) may describe a fall in the presence of air resistance and upthrust

   B) Curves (ii) and (iii) may describe a fall in the presence of air resistance and upthrust

   C) Only curve (ii) may describe a fall in the presence of air resistance but without upthrust

   D) Only curve (iii) may describe a fall in the presence of air resistance but without upthrust

   E) All of the curves (i)–(iii) may describe a fall in the presence of air resistance and upthrust

   Figure 1c所示之曲線(i)–(iii)描述一落體（離地表沒有太遠）之速度 v 隨時間 t 的變化情形，且直線(i)之斜率等於 $g = 9.8$ m/s^2。下列敘述何者正確？

   A) 曲線(i)–(iii)皆不可能描述有空氣阻力與浮力之落體

   B) 曲線(ii)和(iii)可能描述有空氣阻力與浮力之落體

   C) 只有曲線(ii)可能描述有空氣阻力但無浮力之落體

   D) 只有曲線(iii)可能描述有空氣阻力但無浮力之落體

   E) 曲線(i)–(iii)皆可能描述有空氣阻力與浮力之落體

5. A mass block attached to a spring on a frictionless ground is released from rest at position $x = A$. When the spring is at its natural length, the mass block is at position $x = 0$, as sketched in Figure 1d (left). About the energy curves sketched in Figure 1d (right), which of the following is correct?

   A) The fact that (iii) is horizontal indicates the conservation of linear momentum

   B) The solid straight line (iii) describes the mechanical energy

   C) The dashed line of curve (ii) describes the potential energy

   D) The dotted line of curve (i) describes the kinetic energy

   E) The curves (i) and (ii) intersect at $x = 0.8A$

   一質量繫於一彈簧在無摩擦之表面上運動，且於位置 $x = A$ 由靜止釋放。當彈簧處於自然長度時，該質量的位置在 $x = 0$，如 Figure 1d(左)所示。關於 Figure 1d(右)所示之能量曲線，下列何者正確？

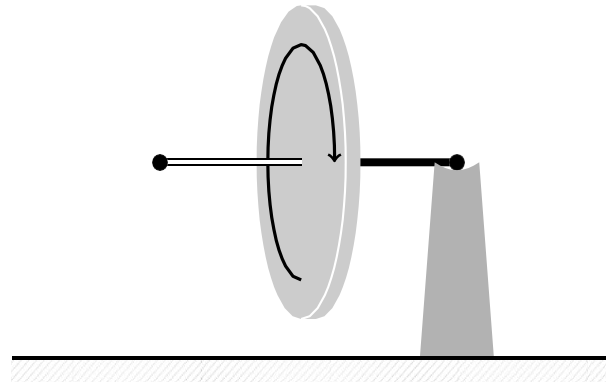   A) (iii)為水平直線所代表意義的是線性動量的守恆

   B) 直線(iii)描述的是機械能

   C) 曲線(ii)描述的是位能
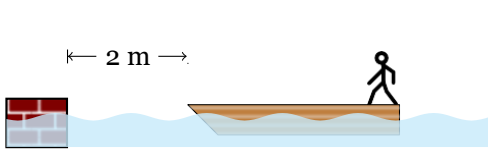
   D) 曲線(i)描述的是動能
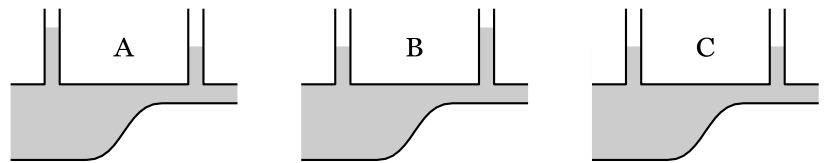
E) 曲線(i)和曲線(ii) 在 $x = 0.8A$ 交叉
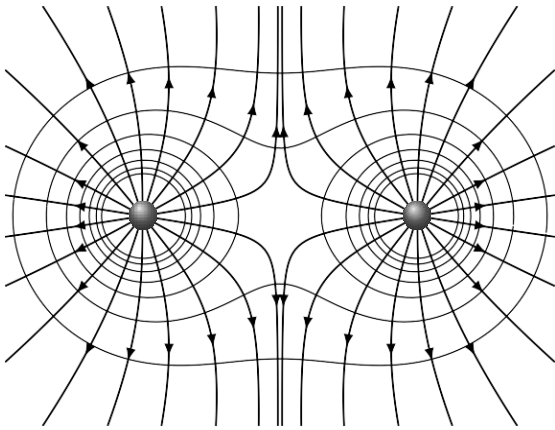
(a) Question 7

(b) Question 8

(c) Question 9

(d) Question 11

Figure 2

6. Continued with 5: The mass undergoes a simple harmonic oscillation with its position as a function of time t given by

$$x(t) = A \cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

with A being the amplitude of the oscillation. Which of the following is correct?

A) The velocity is independent of A
B) The period is independent of A
C) The larger the amplitude A, the longer the period
D) The larger the amplitude A, the shorter the period
E) The period is given by $2\pi\sqrt{k/m}$

續5：該質量進行簡諧運動，其位置隨時間 t 變化的函數為

$$x(t) = A \cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

其中，A是振幅。下列敘述何者正確？

A) 質量運動的速度與A無關
B) 週期與A無關
C) A越大，週期越長
D) A越大，週期越短
E) 週期為$2\pi\sqrt{k/m}$

7. Consider the electric field lines (with arrow heads) and equipotential surface (without arrow heads) shown in Figure 2a. Which of the following is possible (Q and V below are both positive)?

A) Both spheres carry charge $-Q$
B) The two spheres are oppositely charged
C) The electric potential on the left sphere is $-V$ and right sphere $+V$
D) The electric field lines and the corresponding equip0tential surface must be perpendicular to each other every where
E) The electric field strength at the midpoint between the two spheres is strongest

考慮Figure 2a所示的電場線(帶有箭頭)與等位線(不帶箭頭)。下列敘述何者是可能的（以下Q與V 皆為正）？

A) 兩球皆帶$-Q$之電荷
B) 兩球所帶電荷為一正一負
C) 左邊的球電位為$-V$，右邊為$+V$
D) 電場線與其對應的等位線必然處處正交
E) 電場強度在兩球的中間最強

8. Consider the gyroscope with its disk spinning at a rather high angular velocity with the direction indicated by the arrow, as shown in Figure 2b. Which of the following is correct?

   A) Without gravity, the gyroscope can also precess
   B) The spinning wheel will immediately fall down because of the gravity
   C) The heavier the disk, the faster the precession.
   D) The precession direction of the gyroscope is clockwise when looking from above
   E) The gyroscope does not necessarily precess

   考慮Figure 2b所繪陀螺儀，其轉盤正以相當的速度在轉動，轉動方向如箭頭所示。下列敘述何者正確？
   A) 陀螺儀置於無重力的環境也會進動
   B) 重力拉扯的關係，陀螺儀會立刻倒下來
   C) 轉盤越重，進動越快
   D) 陀螺儀進動的方向（從上面看）是順時針
   E) 陀螺儀不一定會進動

9. A 40-kg boy is at the rear of a stationary boat of mass 60 kg and length 3 m, which can move freely on the water, as shown in Figure 2c. The front of the boat is of distance 2 m to the dock initially. When the boy reaches the front end, how far is the boy to the dock?

   A) 2 m   B) 3 m   C) 3.2 m   D) 3.5 m   E) 3.8 m

   一個四十公斤重的小男孩從船的一端試圖走向另一端，如Figure 2c 所示。若船重六十公斤、長三公尺，船可在水面上無摩擦滑動，且開始走之前，船頭距離碼頭兩公尺遠，試問小男孩走到船頭時，距離碼頭有多遠？

   A) 2 m   B) 3 m   C) 3.2 m   D) 3.5 m   E) 3.8 m

10. About conservation of linear momentum (LM), which of the following is correct?

   A) Two objects collide with each other in the absence of external forces. No matter the collision is elastic or inelastic, the total LM of the two objects is always conserved
   B) Two objects collide with each other in the absence of external forces. The total LM of the two objects is conserved only when the collision is inelastic
   C) Two objects collide with each other in the absence of external forces. The total LM of the two objects is conserved only when the collision is elastic
   D) Whether the total LM of a system of more than two objects is conserved or not depends on the internal forces between the objects
   E) The total LM of a system of more than two objects is never conserved

有關線性動量的守恆，下列敘述何者正確？

A) 兩個物體在無外力的情況下相互碰撞。無論碰撞是彈性或非彈性，兩物體之總線性動量總是守恆的
B) 兩個物體在無外力的情況下相互碰撞。唯有非彈性碰撞，兩物體之總線性動量才會守恆
C) 兩個物體在無外力的情況下相互碰撞。唯有彈性碰撞，兩物體之總線性動量才會守恆
D) 兩個以上的物體，其總線性動量守恆與否，與內力有關
E) 兩個以上的物體，其總線性動量不可能守恆

11. Consider a Venturi tube filled with a flowing liquid, as shown in Figure 2d. Which of the following is correct?

   A) Both of sketches A and B can happen, but not C
   B) All of A, B, and C can happen
   C) Only sketch A makes good sense
   D) Only sketch B makes good sense
   E) Only sketch C makes good sense

   考慮Figure 2d 所繪之文氏管（喉形管），內有流動中的液體。下列何者正確？
   A) A圖和B圖都有可能發生，但C不可能
   B) 三個圖都有可能發生
   C) 只有A圖是合理的
   D) 只有B圖是合理的
   E) 只有C圖是合理的

12. Consider the electric fields E_Q, E_λ, and E_σ due to an isolated point charge Q, an isolated infinitely long line charge density λ, and an isolated infinitely extending plane charge density σ, respectively. Which of the following is correct?

   A) $E_Q \propto r^{-3}$ where r is the distance to the point charge
   B) $E_Q \propto r^{-3/2}$ where r is the distance to the point charge
   C) $E_\lambda \propto r^{-3/2}$ where r is the shortest distance to the line
   D) $E_\lambda \propto r^{-2}$ where r is the shortest distance to the line
   E) E_σ is constant, independent of position
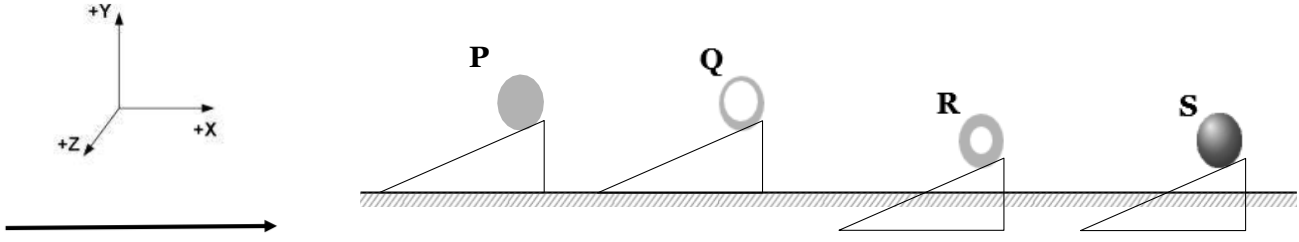
(a) Question 14   (b) Question 15

Figure 3

考慮點電荷Q所產生的電場E_Q、無窮長線電荷密度 λ 所產生的電場 E_λ、以及無窮延伸之面電荷密度 σ 所產生的電場 $E\sigma$。下列何者正確？

A) $E_Q \propto r^{-3}$ ，其中 r 是到 Q 的距離

B) $E_Q \propto r^{-3/2}$ ，其中 r 是到 Q 的距離

C) $E_\lambda \propto r^{-3/2}$ ，其中 r 是到 λ 的最短距離

D) $E_\lambda \propto r^{-2}$ ，其中 r 是到 λ 的最短距離

E) E_σ 是常數，與位置無關

13. A capacitor of capacitance C is made of two conducting plates. Which of the following is correct?

    A) The capacitance doubles when the applied voltage is doubled

    B) The capacitance doubles when the amount of charge stored is doubled

    C) Two such capacitors connected in series have an equivalent capacitance $2C$

    D) Two such capacitors connected in parallel have an equivalent capacitance $C/2$

    E) The capacitance depends neither on the applied voltage nor on the stored charge

一電容器由兩塊導體組成，其電容為C。下列敘述何者正確？

    A) 若施加電壓增倍，則電容值加倍
    B) 若電容器儲存之電荷加倍，電容值亦加倍
    C) 兩個這樣的電容器串聯後，其等效電容為$2C$
    D) 兩個這樣的電容器並聯後，其等效電容為$C/2$
    E) 電容值並不取決於施加電壓及儲存電荷

14. A charged particle travels in a straight line with uniform velocity v in presence of crossed electric field **E** and magnetic field **B**, shown in Figure 3a. Given that **B** = (0, 0, B) with $B > 0$ and v=(v,0,0), what is the electric field **E** ?

    A) (0, -Bv, 0)
    B) (0, Bv, 0)
    C) (0, 0, Bv)
    D) (0, 0, -Bv)
    E) (Bv, 0, 0)

考慮一帶電粒子於交錯電場**E**與磁場**B**的作用下，均速直線運動，呈現如Figure 3a 所示的軌跡。 若磁場為 **B** = (0, 0, B)，$B > 0$， 速度為 v=(v,0,0) ，則電場為何？

    A) (0, -Bv, 0)
    B) (0, Bv, 0)
    C) (0, 0, Bv)
    D) (0, 0, -Bv)
    E) (Bv, 0, 0)

15. A solid cylinder **P**, two hollow cylinders **Q** & **R** (**Q** has thinner shell than **R**), and a solid sphere **S** roll down an inclined plane from the same height without slipping, as sketched in Figure 3b. If all objects have the same mass & radius, without air resistance, what is the sequence from fastest to slowest that they will reach the bottom of the inclined plane?

    A) **Q, R, P, S**

    B) **P, Q, R, S**

    C) **S, P, R, Q**

    D) **S, R, Q, P**

    E) **R, Q, P, S**

一實心圓柱 **P**、兩空心圓柱 **Q** 和 **R**（**Q** 的厚度比 **R** 薄）、以及一實心球 **S**，同時從同樣的高度無滑動滾下，如Figure 3b所示。如全部物體的質量和半徑相同，不考慮空氣阻力的話，下列何者將以最快至最慢的順序先碰到平地？

    A) **Q, R, P, S**
    B) **P, Q, R, S**
    C) **S, P, R, Q**
    D) **S, R, Q, P**
    E) **R, Q, P, S**
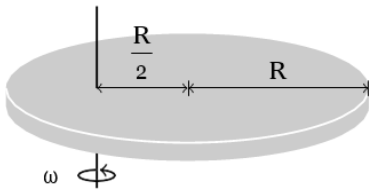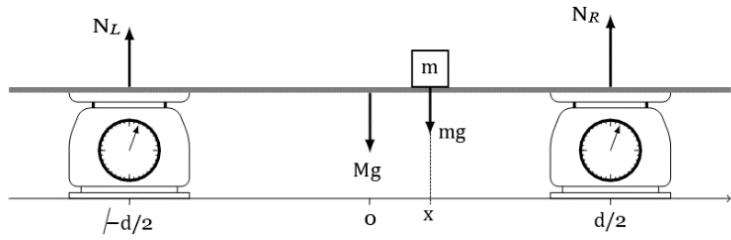
(a) Question 17

(b) Question 19

Figure 4

16. Consider one mole of an ideal gas at $20°C$ and 1 atm. Given the ideal gas constant $R = 8.3 \text{ J K}^{-1}\text{mol}^{-1}$, which of the following is closest to the volume of the above-mentioned gas ?

    A) 20 L   B) 21 L   C) 22 L   D) 23 L   E) 24 L

    考慮一大氣壓、攝氏二十度下的一莫耳理想氣體。若理想氣體常數為 $R = 8.3 \text{ J K}^{-1}\text{mol}^{-1}$，下列選項哪個最接近該氣體的體積？

    A) 20 L   B) 21 L   C) 22 L   D) 23 L   E) 24 L

17. A uniform disk of mass M and radius R is rotating in the way shown in Figure 4a. If the angular momentum of the disk rotating with ω is $\ell MR^2\omega$, what should ℓ be?

    A) 1/4   B) 1/2   C) 3/4   D) 1   E) 5/4

    一均勻圓盤，質量為M、半徑為 R 以Figure 4a所呈現的方式轉動。若轉盤之角速度為ω，角動量為$\ell MR^2\omega$，則ℓ =?

    A) 1/4   B) 1/2   C) 3/4   D) 1   E) 5/4

18. A satellite of mass m_a, named Alpha, is orbiting around the earth. The orbit is approximately a circle with radius r and period T_a. Which of the following is correct?

    A) If another satellite with mass $m = 2m_a$ has the same orbit, its period will be same as T_a
    B) If Alpha is now orbiting around the moon with exactly the same orbit, its period will be same as T_a
    C) If the orbit of Alpha is an ellipse, its orbiting speed will vary in position, and so does its total mechanical energy
    D) $T_a \propto r^3$, consistent with Kepler's third law
    E) $T_a \propto r^{2/3}$, consistent with Kepler's third law

    一具繞地球運動的衛星, 名為阿爾法, 其質量為 m_a。若該軌道為半徑 r 的圓形, 週期 T_a, 下列敘述何者正確？

A) 若有另一具衛星，質量為$m = 2m_a$，以相同的軌道運動，則其週期與 T_a 一樣
B) 若阿爾法衛星以一模一樣的軌道繞月球運動，則其週期與 T_a 一樣
C) 若阿爾法衛星的軌道為橢圓，則其繞軌速率會隨位置改變，總機械能也是
D) 由克普勒第三定律可知，$T_a \propto r^3$
E) 由克普勒第三定律可知，$T_a \propto r^{2/3}$

19. Consider the static equilibrium problem shown in Figure 4b. About N_L and N_R, which of the following is correct?

    A) $N_L = \left(\dfrac{M/m + 1}{2} - \dfrac{x}{d}\right)mg$

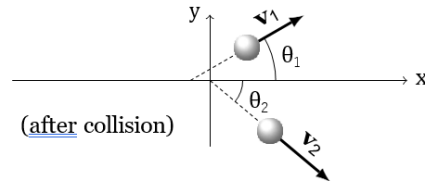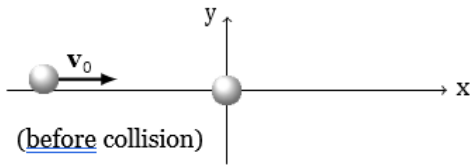   B) $N_R = \left(\dfrac{M/m + 1}{2} - \dfrac{x}{d}\right)mg$

    C) $N_L = \left(\dfrac{M}{m} + 1 + \dfrac{x}{d}\right)mg$
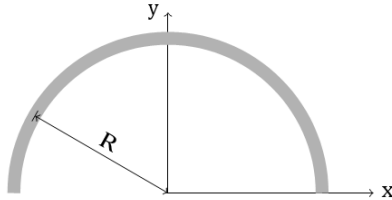
    D) $N_R = \left(\dfrac{M}{m} + 1 + \dfrac{x}{d}\right)mg$

    E) $N_L = N_R = (M + m)g/2, \text{independent of } x$

    考慮如 Figure 4b 所示之靜力平衡問題。關於 N_L 與 N_R，下列何者正確？

    A) $N_L = \left(\dfrac{M/m + 1}{2} - \dfrac{x}{d}\right)mg$

    B) $N_R = \left(\dfrac{M/m + 1}{2} - \dfrac{x}{d}\right)mg$

    C) $N_L = \left(\dfrac{M}{m} + 1 + \dfrac{x}{d}\right)mg$

    D) $N_R = \left(\dfrac{M}{m} + 1 + \dfrac{x}{d}\right)mg$
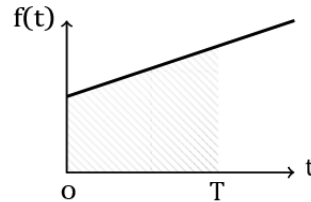
    E) $N_L = N_R = (M + m)g/2$ 總是成立，無關乎 x

(a) Question 20

(b) Question 21

(c) Question 22

Figure 5

20. Two identical balls of mass m undergo a two-dimensional elastic collision, one ball being initially at rest and the other approaching with velocity $\mathbf{v}_0$, as shown in Figure 5a. Which of the following is correct ?
A) Due to linear momentum conservation, $v_0 = v_1 + v_2$
B) Due to linear momentum conservation, $\overrightarrow{v_0} = \overrightarrow{v_1} + \overrightarrow{v_2}$
C) Due to linear momentum conservation, $v_0^2 = v_1^2 + v_2^2$
D) $\theta_1 + \theta_2 \neq 90°$ can happen for such elastic collisions
E) $\theta_1 + \theta_2 = 90°$ is always true, even if the collision is not elastic

兩個一模一樣的球，質量為 m，進行二維的彈性碰撞，一個原處於靜止，另一個以速度 $\mathbf{v}_0$ 接近，如 Figure 5a 所示。下列敘述何者正確？

A) 由於線性動量的守恆，所以 $v_0 = v_1 + v_2$
B) 由於線性動量的守恆，所以 $\overrightarrow{v_0} = \overrightarrow{v_1} + \overrightarrow{v_2}$
C) 由於線性動量的守恆，所以 $v_0^2 = v_1^2 + v_2^2$
D) 對於這樣的彈性碰撞，$\theta_1 + \theta_2 \neq 90°$ 也可能發生
E) 即使碰撞是非彈性，$\theta_1 + \theta_2 = 90°$ 也必然為真

21. A semicircular ring of radius R and uniform mass density is shown in Figure 5b. If the coordinate of the center of mass is $(0, y_{cm})$, what should y_{cm} be?

A) $\dfrac{2R}{5\pi}$　　B) $\dfrac{R}{5\pi}$　　C) $\dfrac{2R}{3\pi}$　　D) $\dfrac{R}{\pi}$　　E) $\dfrac{2R}{\pi}$

一半圓環半徑為 R 且質量分布均勻，如 Figure 5b 所示。若其質心座標為 $(0, y_{cm})$，則 y_{cm} 應為？

A) $\dfrac{2R}{5\pi}$　　B) $\dfrac{R}{5\pi}$　　C) $\dfrac{2R}{3\pi}$　　D) $\dfrac{R}{\pi}$　　E) $\dfrac{2R}{\pi}$

22. An object undergoes a motion in a straight line with f as a function of time t sketched in Figure 5c. Which of the following is **incorrect**?

A) If f is its velocity, the shaded area represents the distance travelled from time 0 to T
B) If f is its position, the net force acting on the object is zero
C) If f is its velocity, the object is undergoing a uniform acceleration
D) If f is its acceleration, the shaded area represents the distance travelled from time 0 to T
E) If f is its position, the velocity remains constant

直線運動中的物體，其 f 隨時間 t 的變化如 Figure 5c 所示。下列敘述何者不正確？

A) 若 f 為速度，則所標示的陰影面積代表的是時間 0 到 T 物體所移動的距離
B) 若 f 為位置，則物體所受淨力為零
C) 若 f 為速度，則物體所進行的是等加速度運動
D) 若 f 為加速度，則所標示的陰影面積代表的是時間 0 到 T 所移動的距離
E) 若 f 為位置，則速度保持定值

23. A ballistic pendulum is shown in Figure 6a. If m = 10g , M = 2 kg, H = 5 cm, what is the initial speed v of the bullet?

A) $100\frac{m}{s}$　　B) $150\frac{m}{s}$　　C) $200\frac{m}{s}$　　D) $250\frac{m}{s}$　　E) $300\frac{m}{s}$

一個彈道擺如 Figure 6a 所示。若 m = 10 g、M = 2kg、H = 5 cm，子彈的初速度 v 是多少？

A) $100\frac{m}{s}$　　B) $150\frac{m}{s}$　　C) $200\frac{m}{s}$　　D) $250\frac{m}{s}$　　E) $300\frac{m}{s}$

24. Consider the circuit shown in Figure 6b. What is the equivalent resistance between point 1 and point 2?

A) $\dfrac{R}{2}$　　B) R　　C) $\dfrac{3R}{2}$　　D) 2R　　E) $\dfrac{5R}{2}$
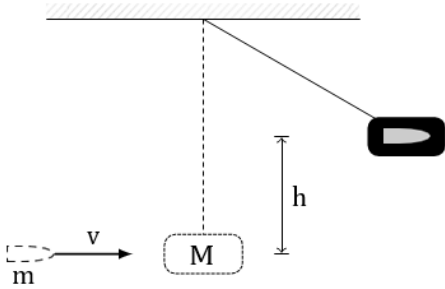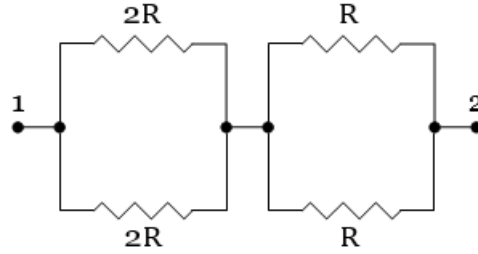
(a) Question 23

Figure 6

考慮如 Figure 6b 所示之電路圖。試問點 1 與點 2 之間的等校電阻為？

A) $\frac{R}{2}$ B) R   C) $\frac{3R}{2}$ D) 2R   E) $\frac{5R}{2}$

25. A copper wire has a length of 1.5 m and a radius of 0.5 mm. Given the Young's modulus $Y = 1.4 \times 10^{11}$ N/m^2, what is the change of its length ΔL when the wire is stretched by a force of 2000 N from one end, while the other end being fixed? Choose the closest number (in units of *cm*)

A) 3.3   B) 3.0   C) 2.7   D) 2.4   E) 2.1

一銅線長度為 1.5 m、半徑為 0.5 mm、楊格係數為 $Y = 1.4 \times 10^{11}$ N/m2。當其一端固定，另一端受 2000N 的拉扯力，試問其伸長量 ΔL 為多少？選擇最接近的數字（單位為公分）。

A) 3.3   B) 3.0   C) 2.7   D) 2.4   E) 2.1

(b) Question 24