

2015年第32屆全國華文獨中數理學識比賽 — 數學

- 考慮三點 $P(1, -1)$, $Q(4, 3)$, $R(a, 2)$ 。若直線 PQ 與直線 PR 垂直，則 a 的值為何？
Consider three points $P(1, -1)$, $Q(4, 3)$, and $R(a, 2)$. If the line PQ is perpendicular to the line PR , then what is the value of a ?
 - 1
 - 1
 - 3
 - 3
 - 以上皆非 None of the above
- 點 $(-1, 2)$ 到直線 $3x + 4y = 15$ 的距離為多少？
What is the distance from the point $(-1, 2)$ to the line $3x + 4y = 15$?
 - 4
 - 2
 - $\frac{2}{5}$
 - $\frac{4}{25}$
 - 以上皆非 None of the above
- 令 P 為圓 $3x^2 + 3y^2 - 4x + 2y = 1$ 的圓心。下列敘述何者正確？
Let P be the center of the circle $3x^2 + 3y^2 - 4x + 2y = 1$. Which of the following statements is true?
 - P 在直線 $y = 2x$ 上。
 P lies on the line $y = 2x$.
 - p 在直線 $3y = x - 1$ 上。
 P lies on the line $3y = x - 1$.
 - P 在拋物線 $y = 1 - 3x^2$ 上。
 P lies on the parabola $y = 1 - 3x^2$.
 - P 在拋物線 $3y = -x^2$ 上。
 P lies on the parabola $3y = -x^2$.
 - P 在圓 $x^2 + y^2 - x - y = 1$ 上。
 P lies on the circle $x^2 + y^2 - x - y = 1$.
- 滿足不等式 $2^n \leq 64 \cdot 2^{\frac{6}{n-1}}$ 的正整數 n 共有多少個？
How many positive integers n satisfying the inequality $2^n \leq 64 \cdot 2^{\frac{6}{n-1}}$?
 - 6
 - 7
 - 8
 - 9
 - 以上皆非 None of the above

5. 共有多少個正整數對 (m, n) 滿足 $m < n$ 與

$$\log_9 m + \log_9 n - \log_9(m + n) = 1?$$

How many pairs (m, n) of positive integers are there such that $m < n$ and

$$\log_9 m + \log_9 n - \log_9(m + n) = 1?$$

- (A) 0
(B) 1
(C) 2
(D) 3
(E) 以上皆非 None of the above
6. 設 m, n 分別為整數 307^{555} 的十位數與個位數，則 $3m + 2n$ 的值為何?
Let m and n be the tens digit and the units digit, respectively, of the integer 307^{555} . What is the value of $3m + 2n$?
- (A) 49
(B) 30
(C) 18
(D) 14
(E) 2
7. 三邊長分別為 7, 12, 13 的三角形的面積為多少?
What is the area of a triangle with side lengths 7, 12, and 13?
- (A) 32
(B) 16
(C) 13
(D) $24\sqrt{3}$
(E) $12\sqrt{3}$
8. 在 $\triangle ABC$ 中，設 a, b, c 分別為角 A, B, C 的對邊。已知 $\triangle ABC$ 的面積為 1 ， $a = 2$ 且 $B = 45^\circ$ ，則 $\frac{a - 2b + 3c}{\sin A - 2\sin B + 3\sin C}$ 的值為何?

In $\triangle ABC$, let a, b , and c be the sides opposite the angles A, B , and C , respectively. Suppose the area of $\triangle ABC$ is 1, $a = 2$, and $B = 45^\circ$. What is the value of $\frac{a - 2b + 3c}{\sin A - 2\sin B + 3\sin C}$?

- (A) 1
(B) 2
(C) $\frac{\sqrt{2}}{2}$
(D) $\sqrt{2}$
(E) $2\sqrt{2}$

9. 若矩陣 $\begin{pmatrix} -1 & -x & 1-x^2 \\ 0 & 1 & -x \\ x & 0 & -x \end{pmatrix}$ 的逆矩陣不存在，則 x 的值為何？

If the matrix $\begin{pmatrix} -1 & -x & 1-x^2 \\ 0 & 1 & -x \\ x & 0 & -x \end{pmatrix}$ has no inverse, then what is the value of x ?

- (A) 1
- (B) -1
- (C) 2
- (D) -2
- (E) 以上皆非 None of the above

10. 已知 $\{a_n\}$ 為等差數列，公差為 3 且 $a_1 = 6$ ，則 $\sum_{k=1}^{32} \frac{1}{a_k a_{k+1}}$ 的值為何？

Let $\{a_n\}$ be an arithmetic sequence with common difference 3 and $a_1 = 6$. What is $\sum_{k=1}^{32} \frac{1}{a_k a_{k+1}}$?

- (A) $\frac{1}{2}$
- (B) $\frac{31}{594}$
- (C) $\frac{8}{17}$
- (D) $\frac{8}{153}$
- (E) 以上皆非 None of the above

11. 設 $\{a_n\}$ 為等比數列，其公比為大於 1 的實數。若 $a_3 + a_6 = -9$ 且 $a_4 a_5 = 8$ ，則 $a_2 a_{10}$ 的值為何？
Let $\{a_n\}$ be a geometric sequence whose common ratio is a real number greater than 1. Suppose $a_3 + a_6 = -9$ and $a_4 a_5 = 8$. What is $a_2 a_{10}$?

- (A) 2
- (B) 256
- (C) 16
- (D) 128
- (E) 64

12. 對 $\theta \in [0, \frac{\pi}{4})$ ， $\frac{\cos \theta \tan 2\theta}{1 + \sec 2\theta}$ 等於下列哪一個選項？

For $\theta \in [0, \frac{\pi}{4})$, which of the following is equal to $\frac{\cos \theta \tan 2\theta}{1 + \sec 2\theta}$?

- (A) $\sin \theta$
- (B) $\cos \theta$
- (C) $\tan \theta$
- (D) $\sec \theta$
- (E) 以上皆非 None of the above

13. 若 $\sin x + \cos y = \frac{3}{2}$ 且 $\cos x + \sin y = \frac{1}{3}$ ，則 $\cos(x - y)$ 的值為何？
 Suppose $\sin x + \cos y = \frac{3}{2}$ and $\cos x + \sin y = \frac{1}{3}$. What is $\cos(x - y)$?
- (A) $\frac{43}{96}$
 (B) $\frac{36}{85}$
 (C) $\frac{13}{72}$
 (D) $\frac{1}{2}$
 (E) 以上皆非 None of the above
14. 設 $p(x)$ 為實係數多項式。已知 $p(x)$ 除以 $x(x - 1)$ 的餘式為 $3x - 1$ ，且 $p(x)$ 除以 $x + 1$ 的餘式為 2 。令 $r(x)$ 為 $p(x)$ 除以 $x(x^2 - 1)$ 的餘式。下列選項何者正確？
 Let $p(x)$ be a polynomial with real coefficients. When $p(x)$ is divided by $x(x - 1)$, the remainder is $3x - 1$, and when $p(x)$ is divided by $x + 1$, the remainder is 2 . Let $r(x)$ be the remainder obtained by dividing $p(x)$ by $x(x^2 - 1)$. Which of the following is true?
- (A) $r(-1) < -3$
 (B) $r(2) > 9$
 (C) $r(-2) < -1$
 (D) $-27 < r(3) < -9$
 (E) $r(-3) > 27$
15. 將「MALAYSIA」一字的字母重新排列，相同字母不相鄰的排法有多少種？
 In how many ways can the letters in "MALAYSIA" be arranged so that no two identical letters are adjacent?
- (A) 720
 (B) 1680
 (C) 2400
 (D) 5040
 (E) 6720
16. 在一場籃球比賽中，詹姆士得到兩次罰球機會。已知他第一罰的命中率為70%，第二罰的命中率為74%。在第一罰投進的情況下，第二罰的命中率為80%。請問他兩罰至少一中的概率為何？
 In a basketball game, James is awarded two free throws. He has a 70% success rate of making the first free throw and a 74% success rate of making the second shot. He has an 80% success rate of making the second free throw, given that he was successful in his first shot. What is the probability that he makes at least one of the two free throws?
- (A) 82%
 (B) 84%
 (C) 86%
 (D) 88%
 (E) 90%

17. 令 $f_0(x) = \frac{x}{x+1}$ 。對任意正整數 n ，令 $f_n(x) = f_0(f_{n-1}(x))$ 。請問 $|f_{2015}^{-1}(2)|$ 的值為何？
 Let $f_0(x) = \frac{x}{x+1}$ and $f_n(x) = f_0(f_{n-1}(x))$ for all positive integers n . What is the value of $|f_{2015}^{-1}(2)|$?
- (A) $\frac{1}{2015}$
 (B) $\frac{1}{2017}$
 (C) $\frac{2}{4031}$
 (D) $\frac{2}{4033}$
 (E) 以上皆非 None of the above
18. 若 α, β 為實數且滿足 $\lim_{n \rightarrow \infty} \frac{(2\alpha - \beta)n^3 + \beta n^2 + 5n + 1}{2n^2 + n + 1} = 2$ ，則 $\alpha^2 + \beta^2$ 的值為何？
 If α and β are real numbers such that $\lim_{n \rightarrow \infty} \frac{(2\alpha - \beta)n^3 + \beta n^2 + 5n + 1}{2n^2 + n + 1} = 2$, then what is $\alpha^2 + \beta^2$?
- (A) 16
 (B) 4
 (C) 24
 (D) 8
 (E) 20
19. 設 $f(x) = (2x)^{\cos(\pi x)}$ 。請問下列哪一個選項等於 $f'(1)$ ？
 Let $f(x) = (2x)^{\cos(\pi x)}$. Which of the following is equal to $f'(1)$?
- (A) $-\frac{1}{2}$
 (B) $-\frac{1}{4}$
 (C) $\frac{1}{2}$
 (D) $\frac{1}{4}$
 (E) 以上皆非 None of the above
20. 若直線 $y - x = 1 + a$ 與拋物線 $y = x^2 - x + 1$ 相切，則 a 的值為何？
 If the line $y - x = 1 + a$ is tangent to the parabola $y = x^2 - x + 1$, then what is a ?
- (A) -1
 (B) -2
 (C) 0
 (D) 1
 (E) 2

21. 函數 $f(x) = 3x^4 - 4x^3 - 12x^2 + 36$ 在閉區間 $\left[-2, \frac{1}{2}\right]$ 上的最小值為何?

What is the smallest value of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 36$ on the closed interval $\left[-2, \frac{1}{2}\right]$?

- (A) 4
- (B) 31
- (C) 17
- (D) 36
- (E) 以上皆非 None of the above

22. $\int_{-2}^2 \frac{5x^7 + 2x^3}{x^6 + 3x^2 + 1} dx =$

- (A) $\frac{2}{3}$
- (B) $\frac{5}{3}$
- (C) 2
- (D) 5
- (E) 以上皆非 None of the above

23. 設複數 ξ 滿足 $\xi^5 = 1$ 以及 $\xi \neq 1$ 。下列選項何者正確?

Let ξ be a complex number such that $\xi^5 = 1$ and $\xi \neq 1$. Which of the following is true?

- (A) $\xi^3 + \xi^2 + \xi = -1$
- (B) $\xi^4 - \xi^3 + \xi^2 - \xi = 1$
- (C) $\xi^5 + 3\xi^4 + 3\xi^3 + 3\xi^2 + 3\xi = -2$
- (D) $\xi^6 - 2\xi^5 - \xi = 2$
- (E) $(\xi^3 + \xi^2 + \xi + 1)(\xi^3 - \xi^2 - \xi + 1) = 0$

24. 設 $\alpha, \beta, \gamma, \delta$ 為方程式 $x^4 - 6x^3 + 18x^2 - 24x + 16 = 0$ 的四根, 則 $\frac{1}{1+\alpha} + \frac{1}{1+\beta} + \frac{1}{1+\gamma} + \frac{1}{1+\delta}$ 的值為何?

Let $\alpha, \beta, \gamma, \delta$ be the four roots of the equation $x^4 - 6x^3 + 18x^2 - 24x + 16 = 0$. What is the value of $\frac{1}{1+\alpha} + \frac{1}{1+\beta} + \frac{1}{1+\gamma} + \frac{1}{1+\delta}$?

- (A) $\frac{3}{2}$
- (B) $\frac{2}{5}$
- (C) $\frac{25}{48}$
- (D) $\frac{82}{65}$
- (E) $\frac{402}{493}$

$$25. \sum_{k=0}^{50} \frac{(-1)^{k+1} 3^k}{(2k)!(100-2k)!} =$$

(A) $\frac{3^{98} - 2^{99}}{100!}$

(B) $\frac{2^{97}}{25 \cdot 99!}$

(C) $\frac{3^{98}}{5 \cdot 99!}$

(D) $\frac{6^{48}}{5 \cdot 101!}$

(E) 以上皆非 None of the above