

1. 把 2, 0, 1 和 9 四個數字放在 a^{bcd} 或 ab^{cd} 的四個位置, 每個數字只能放在一個位置, 請問最大的數字為何?

What is the maximum number if we put 2, 0, 1, and 9 at the four locations of a^{bcd} or ab^{cd} ? (one digit can be used only once at one location)

- (a) 2^{910}
 (b) 9^{210}
 (c) 91^{20}
 (d) 20^{91}
 (e) 以上皆非。None of the above.

2. 把 $a_1 = 2019$ 的四個數字能排出的最大數字 9210 減去最小數字 0129, 會得到 $a_2 = 9081$, 再把 a_2 的四個數字能排出的最大數字 9810 減去最小數字 0189, 會得到 $a_3 = 9621$, 依此規則, 求 a_{2019} 的值。

For $a_1 = 2019$, we rearrange the digits of a_1 to get the largest number and smallest numbers, then the smallest number, 0129, is subtracted from the largest number, 9210, to get a new number $a_2 = 9081$. Then repeat the operation for each new number, for example, $a_3 = 9810 - 0189 = 9621$. Find the value of a_{2019} .

- (a) 8352
 (b) 6174
 (c) 0
 (d) 1024
 (e) 以上皆非。None of the above.

3. $f(x) = \sum_{k=1}^{2019} \frac{1}{k(k+x)}$, $f(1) = ?$

- (a) 1
 (b) $\frac{2018}{2019}$
 (c) $\frac{2020}{2019}$
 (d) $\frac{2019}{2020}$
 (e) 以上皆非。None of the above.

4. n 是一正整數, $n \leq 2019$, 且 n 與 2019 互質, 請問這樣的 n 有幾個?
How many positive integers, n , where $n \leq 2019$ that satisfy the condition that n and 2019 are co-prime?
- (a) 2018
(b) 1346
(c) 1314
(d) 673
(e) 以上皆非。None of the above.
5. 以下哪一個式子可以算出 2019?
Which of the following expression will yield 2019?
- (a) $1 + 2 \times 3 \times 4 + 5 \times 6 \times (7 \times 8 + 9)$
(b) $1 + 2 \times 34 + 5 \times 6 + (7 \times 8 + 9)$
(c) $1 + 2 \times 3 \times 4 + 5 \times 6 \times (7 + 8 \times 9)$
(d) $1 + 2 \times 34 + 5 \times 6 \times (7 \times 8 + 9)$
(e) 以上皆非。None of the above.
6. 對兩函數 $f(x) = 1.4^x$ 與 $g(x) = \log_{1.4} x$ 的敘述何者正確?
Which of the following description for $f(x) = 1.4^x$ and $g(x) = \log_{1.4} x$ is correct?
- (a) $f(2) > g(2)$
(b) $f(1.4^6) < g(1.4^6)$
(c) 兩函數至少有兩個交點。There are at least two intersections for the two functions.
(d) 兩函數沒有交點。There is no intersection for the two functions.
(e) 以上皆非。None of the above.

7. 對於 $f(x) = x(x - 1) - 3x(x - 2) + 2(x - 1)(x - 2)$ 的敘述, 下列何者正確?
Which of the following description for $f(x) = x(x - 1) - 3x(x - 2) + 2(x - 1)(x - 2)$ is correct?
- (a) $f(x)$ 化簡後是二次多項式。 $f(x)$ is a polynomial of degree 2 after simplification.
- (b) $f'(x) + 1 = 0$
- (c) $f(3) = 4$
- (d) $f(x)$ 除以 $(x - 2)$ 的餘式是 1。 The remainder of $f(x)$ is 1 when divided by $(x - 2)$.
- (e) 以上皆非。 None of the above.
8. 兩個多項式 $f(x)$ 及 $g(x)$, $f(x)$ 除以 $g(x)$ 的餘式為 $(x - 1)$, 而 $g(x)$ 除以 $f(x)$ 的餘式也是 $(x - 1)$, 下列敘述何者正確?
For two polynomials $f(x)$ and $g(x)$, $f(x)$ has a remainder of $(x - 1)$ when divided by $g(x)$ and $g(x)$ also has a remainder of $(x - 1)$ when divided by $f(x)$. Which of the following is correct?
- (a) $f(x) + g(x) = x - 1$
- (b) $f(x) - g(x) = x - 1$
- (c) $f(x)$ 是二次多項式。 $f(x)$ is a polynomial of degree 2.
- (d) $f(x)$ 跟 $g(x)$ 的次數不同。 The degrees of $f(x)$ and $g(x)$ are different.
- (e) 以上皆非。 None of the above.
9. 考慮所有的 $a, b \in \mathbb{R}$, 對於函數 $y = f(x) = a \log_2 x + b$ 的敘述何者正確?
Which of the following description for the function $y = f(x) = a \log_2 x + b$ is correct by considering all $a, b \in \mathbb{R}$?
- (a) $y = f(x)$ 不可能是一條直線。 $y = f(x)$ is never a straight line.
- (b) $\lim_{x \rightarrow \infty} f(x)$ 一定不存在。 $\lim_{x \rightarrow \infty} f(x)$ never exists.
- (c) 可以找到 a, b 使得沒有整數點 (m, n) , $m, n \in \mathbb{Z}$, 能滿足 $n = f(m)$ 。 There exists a, b which makes no integer point (m, n) , $m, n \in \mathbb{Z}$, that satisfies $n = f(m)$.
- (d) $y = f(x)$ 跟 $2^y = b \cdot x^a$ 的圖形在 $x > 0$ 一定會完全重合。 The graph of $y = f(x)$ are the same as the graph of $2^y = b \cdot x^a$ when $x > 0$.
- (e) 以上皆非。 None of the above.

10. 四條直線 $\Gamma_1: ax - y = m$, $\Gamma_2: ax - y = n$, $\Gamma_3: x + ay = m$, $\Gamma_4: x + ay = n$, 所圍出的平行四邊形面積為?

What is the area bounded by the four lines $\Gamma_1: ax - y = m$, $\Gamma_2: ax - y = n$, $\Gamma_3: x + ay = m$, $\Gamma_4: x + ay = n$?

(a) $\frac{(m-n)^2}{\sqrt{a^2+1}}$

(b) $\frac{(m-n)^2}{a^2+1}$

(c) $\frac{(m-n)^2}{2a^2+2}$

(d) $\frac{(m-n)^2}{2\sqrt{a^2+1}}$

(e) 以上皆非。None of the above.

11. $(1 + i)^{2019} = ?$

(a) $2^{1009}(1 + i)$

(b) $2^{1009}(1 - i)$

(c) $2^{1009}(-1 - i)$

(d) $2^{1009}(-1 + i)$

(e) 以上皆非。None of the above

12. 方程式 $x^4 + (m - 5)x^2 + m + 3 = 0$ 有四個相異實根, 則 m 可以為下列何數?

The equation $x^4 + (m - 5)x^2 + m + 3 = 0$ has four different real roots, then $m = ?$

(a) -3

(b) -1

(c) 1

(d) 3

(e) 以上皆非。None of the above.

13. $\triangle ABC$, $\overline{AB} = 3$, $\overline{BC} = 6$, $30^\circ \leq \angle B \leq 120^\circ$, 請問 $\angle C$ 的角度最大?
 $\triangle ABC$, $\overline{AB} = 3$, $\overline{BC} = 6$, $30^\circ \leq \angle B \leq 120^\circ$, What is the maximum value of $\angle C$?
- (a) 15°
 (b) 30°
 (c) 45°
 (d) 60°
 (e) 以上皆非。 None of the above.
14. P 在 $\triangle ABC$ 中, $\triangle PBC : \triangle PAC : \triangle PAB = 1 : 2 : 3$. $\overline{OP} = \alpha \overline{OA} + \beta \overline{OB} + \gamma \overline{OC}$,
 $(\alpha, \beta, \gamma) = ?$
 P is inside $\triangle ABC$, $\triangle PBC : \triangle PAC : \triangle PAB = 1 : 2 : 3$. $\overline{OP} = \alpha \overline{OA} + \beta \overline{OB} + \gamma \overline{OC}$,
 $(\alpha, \beta, \gamma) = ?$
- (a) (1, 2, 3)
 (b) $(\frac{1}{6}, \frac{1}{3}, \frac{1}{2})$
 (c) 6 : 3 : 2
 (d) $(\frac{6}{11}, \frac{3}{11}, \frac{2}{11})$
 (e) 以上皆非。 None of the above.
15. Alice、Bob、Cathy 輪流不停地 ($A \rightarrow B \rightarrow C \rightarrow A \rightarrow B \rightarrow \dots$) 丟擲三粒公正骰子,
 先丟出點數和為 10 點者獲勝後停止, 則此三人獲勝機率的比值為下列何者?
 Alice, Bob and Cathy take turns to roll three fair dices ($A \rightarrow B \rightarrow C \rightarrow A \rightarrow B \rightarrow \dots$).
 The one who first rolls total 10 wins the game and the game stops. What is the ratio of the
 winning probabilities of Alice, Bob, and Cathy?
- (a) 1 : 1 : 1
 (b) 64 : 8 : 1
 (c) 27 : 14 : 13
 (d) 64 : 56 : 49
 (e) 以上皆非。 None of the above.

16. $a \in \mathbb{N}$, $[a, 20] = 7a + 1314$, $[a, 20]$ 表示 a 與 20 的最小公倍數, $a = ?$
 $a \in \mathbb{N}$, $[a, 20] = 7a + 1314$, $[a, 20]$ stands for the least common multiple of a and 20, $a = ?$

- (a) 78
 (b) 438
 (c) 1314
 (d) 2019
 (e) 以上皆非。None of the above.

17. $\triangle ABC$ 中, \overline{BC} 的中垂線交 \overline{AC} 於 D 點, 且 \overline{BD} 是 $\angle ABC$ 的角平分線. 若 $\overline{AD} = 8$, $\overline{CD} = 10$, $\triangle ABD$ 的面積為何?

In a triangle $\triangle ABC$, the side \overline{AC} and the perpendicular bisector of \overline{BC} meet at point D , and \overline{BD} bisects $\angle ABC$. If $\overline{AD} = 8$ and $\overline{CD} = 10$, what is the area of $\triangle ABD$?

- (a) $15\sqrt{2}$
 (b) $15\sqrt{3}$
 (c) $15\sqrt{5}$
 (d) $15\sqrt{7}$
 (e) 以上皆非。None of the above.

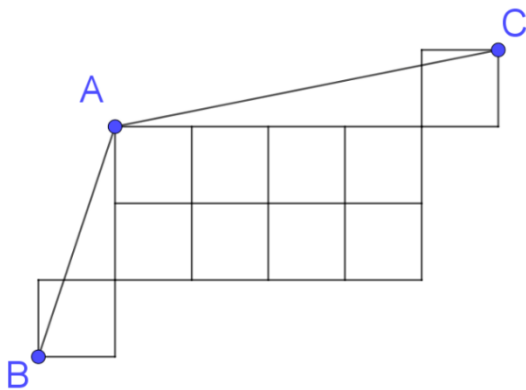
18. $n \in \mathbb{N}$, $\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}^n = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix}$, 下列敘述何者正確?
 $n \in \mathbb{N}$, $\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}^n = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix}$, which of the following is correct?

- (a) $a_3 = 9$
 (b) $a_2 + a_3 = a_4$
 (c) $a_4 + b_4 = c_4 + d_4$
 (d) $c_n + d_n = 3$
 (e) 以上皆非。None of the above.

19. $\lim_{x \rightarrow a} \frac{x^2 - a^2}{\sqrt{x} - \sqrt{a}} = ?$

- (a) $2a$
- (b) $2a\sqrt{a}$
- (c) $4a\sqrt{a}$
- (d) $2\sqrt{a}$
- (e) 以上皆非。 None of the above.

20. 下圖是由 10 個小正方形拼成的，其中 A 、 B 、 C 三點如圖所示，求 $\tan \angle BAC = ?$
 The following figure is made by 10 squares. The points A , B , and C are shown in figure.
 Find $\tan \angle BAC$.



- (a) -1.75
- (b) -1.5
- (c) -1
- (d) $-\sqrt{3}$
- (e) 以上皆非。 None of the above.

21. 關於 $f(x) = x^3 + ax + 1$ 的敘述, 下列何者正確?

Which of the following description about $f(x) = x^3 + ax + 1$ is correct?

- (a) $a > 0$ 時, $f(x)$ 只有一正實根。When $a > 0$, $f(x)$ only has one positive real root.
- (b) $a < -1$ 時, $f(x)$ 必有三個實根。When $a < -1$, $f(x)$ has three real roots.
- (c) 不管 a 是什麼數, 都只有一條 $f(x)$ 的切線可以通過 $(0, 1)$ 。There is only one tangent line of $f(x)$ which passes through $(0, 1)$ for any real number a .
- (d) 可以找到 a 使得 $f(x)$ 可以有兩個負根, 一個正根。There exists a such that $f(x)$ can have two negative roots and one positive root.
- (e) 以上皆非。None of the above.

22. $f(x) = \int_0^t t^2 x^2 + 2tx + 3 dx$. $f'(t) = ?$

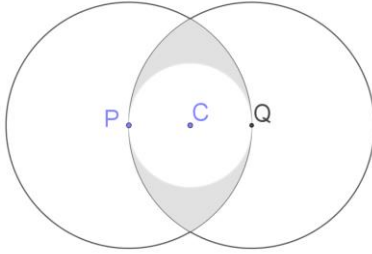
- (a) $\frac{4}{3}t^3 + 2t^2 + 3$
- (b) $\frac{4}{3}t^3 + 3t^2 + 3$
- (c) $\frac{5}{3}t^4 + 2t^2 + 3$
- (d) $\frac{5}{3}t^4 + 3t^2 + 3$
- (e) 以上皆非。None of the above.

23. Alice、Bob、Cathy、Don、Elsa、Fenny 等六人點冰淇淋, 有巧克力、香草、芒果三種口味可選擇, 但每種口味只剩 4 份可提供。若每人選擇一份, 且 Alice 和 Bob 不選同一種, 則這六人選擇的方式有幾種?

Alice, Bob, Cathy, Don, Elsa, and Fenny order three flavours of ice cream, chocolate, vanilla, and mango, but only 4 servings per flavour are available. If each person chooses one and Alice and Bob do not choose the same one, how many ways that the six people can choose their ice cream?

- (a) 432
- (b) 474
- (c) 486
- (d) 729
- (e) 以上皆非。None of the above.

24. 一個半徑為 1 的小圓內切在兩個半徑為 2 的大圓中，切點是 P 跟 Q ，且 PQ 也正是小圓的直徑，如下圖所示，請問在小圓之外，大圓之內的灰色面積是多少？
 A circle of radius 1 is internally tangent to two circles of radius 2 at points P and Q , where PQ is a diameter of the smaller circle. What is the area of the shaded region in the figure, that is outside the smaller circle and inside each of the two larger circles?



- (a) $\frac{5}{3}\pi - 2\sqrt{3}$
 (b) $\frac{2}{3}\pi - \sqrt{3}$
 (c) $\frac{4}{3}\pi - \sqrt{3}$
 (d) $\frac{7}{3}\pi - 2\sqrt{3}$
 (e) 以上皆非。None of the above.
25. 求一直線 $\{(x, y, z) \mid x + y + z = 3, x - y + z = 1\}$ 與原點 $(0, 0, 0)$ 最近的距離。
 Find the shortest distance between the line $\{(x, y, z) \mid x + y + z = 3, x - y + z = 1\}$ and the origin $(0, 0, 0)$.
- (a) 3
 (b) $\sqrt{3}$
 (c) $\sqrt{2}$
 (d) 1
 (e) 以上皆非。None of the above.