1. 關於2018的敘述,下列何者正確?

Which of the following statements about the number 2018 is true?

- (a) 它可以表示成兩個正整數的平方和. There exist a, b  $\in$  N such that  $2018 = a^2 + b^2$ .
- (b) 它可以寫成三個相鄰整數的乘積. There exists  $n \in \mathbb{N}$  such that 2018 = n (n + 1)(n + 2).
- (c) 它可以表示成兩個質數的乘積. 2018 can be expressed by  $p \times q$ , where p and q are primes.
- (d) 它可以寫成兩個階乘的數的差. 2018 can be expressed by m! - n!, where m, n  $\in$  N
- (e) 以上皆非. None of the above.
- 2.  $a = 0.\overline{2}, b = 1.\overline{8}$ , 下列敘述何者正確?

If  $a = 0.\overline{2}$ ,  $b = 1.\overline{8}$ , which of the following statements is true?

- (a) a + b = 2
- (b) b 4a = 1
- (c)  $\frac{b}{a} = 8$
- (d) 9a = b
- (e) 以上皆非. None of the above.
- 3. Alice, Bob 各從2,0,1,8 這四個數字任選(可重複選) 兩個寫成一個二位數a 及 b, 已知Alice 寫的數字比Bob 大, 那麼此時Alice 的個位數比Bob 的個位數大的 機率是?

Alice and Bob choose two numbers (the two numbers can be the same) from 2, 0, 1, and 8 to make a two-digit number a and b, respectively. If the number made by Alice a is greater than Bob's number b, what is the probability that the ones digit of a is also greater than the ones digit of b?

- (a)  $\frac{15}{32}$ . (b)  $\frac{7}{16}$ . (c)  $\frac{1}{2}$ . (d)  $\frac{16}{9}$ . (e)  $\frac{17}{32}$ .

4.  $y = f(x) = \sum_{k=1}^{2018} x(x-k)$ , 當x 是多少時, y 有最小值?

What is x when  $y = f(x) = \sum_{k=1}^{2018} x(x-k)$ , has its minimum?

- (a) 1009.
- (b) 2019.
- (c)  $\frac{1009}{2}$
- (d)  $\frac{2019}{4}$
- (e) 以上皆非. None of the above.
- 5. 在1~2018 中,是2且3的倍數但不是8的倍數的有幾個?

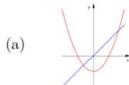
From 1 to 2018, how many numbers have the factors 2 and 3, but do not have the factor 8?

- (a) 250
- (b) 252
- (c) 254
- (d) 256
- (e) 以上皆非. None of the above.
- 6. 方程式xy-2x-3y=1 有幾個整數解?

How many solutions (x, y), x,  $y \in \mathbb{Z}$  are there for the equation xy - 2x - 3y = 1?

- (a) 1.
- (b) 2.
- (c) 4.
- (d) 8.
- (e) 以上皆非. None of the above.
- 7.  $a, b \in \mathbb{R}$ , 下列何者可能是 $y = ax^2 + b$  與y = ax + b 的圖形?

For  $a, b \in \mathbb{R}$ , which of the following figure is the graph for  $y = ax^2 + b$  and y = ax + b?















(e) 以上皆非. None of the above.

8. 多項式f(x) 除以(x-1) 的餘式為3,除以(x-2) 的餘式為2,那麼g(x) = (x+1) f(x) 除以(x-1)(x-2) 的餘式為?

For the polynomial f(x), polynomial division of f(x) by (x-1) and (x-2) gives the remainder 3 and 2, respectively. What is the remainders for the polynomial division of g(x) = (x+1) f(x) by (x-1)(x-2)?

- (a) 6.
- (b) x + 4.
- (c) x + 5.
- (d) 2x + 4.
- (e) 以上皆非. None of the above.
- 9. 多項式f(x) = x(x-2)(x-4) + ax(x-2) 8x + 6,  $a \in \mathbb{R}$ , f(4) > 0, 下列敘述何者正確? For the polynomial f(x) = x(x-2)(x-4) + ax(x-2) 8x + 6,  $a \in \mathbb{R}$ , f(4) > 0, which of the following statements is true?
  - (a) f(x) = 0 有大於4 的根. f(x) = 0 has one root which is greater than 4.
  - (b) 2+i 是 f(x)=0 的根. 2+i is one root of f(x)=0, where  $i=\sqrt{-1}$ .
  - (c) f(x) 沒有負根. f(x) = 0 has no negative root.
  - (d) f(x) 與 $y = -x^2$  只有一個交點. There is only one intersection of f(x) and  $y = -x^2$ .
  - (e) 以上皆非. None of the above.
- 10. 滿足不等式 $4x < 3x^3 3x + 2 < |4x|$ 中的x 可以是下列哪個值? Which x can satisfy the inequality  $4x < 3x^3 3x + 2 < |4x|$ ?
  - (a) 3
  - (b) 0
  - (c) -0.7
  - (d)  $-\sqrt{3}$
  - (e) 以上皆非. None of the above.
- 11. 26! 是幾位數? How many digits are there in (26!)? (log10 28!  $\approx$  29.4841, 26! = 1 x 2 x 3 x … x 26, log10 2  $\approx$  0.301, log10 3  $\approx$  0.4771, log10 7  $\approx$  0.8451)
  - (a) 25.
  - (b) 26.
  - (c) 27.
  - (d) 28.
  - (e) 以上皆非. None of the above.

12. L: x + y = 20 與四條曲線 $\Gamma 1: y = 2^x$ ,  $\Gamma 2: y = 3^x$ ,  $\Gamma 3: y = \log_2 x$ ,  $\Gamma 4: y = \log_3 x$  分別相交於A = (x1; y1), B = (x2; y2), C = (x3; y3), D = (x4; y4) 四點,下列敘述何者正確?

 $A=(x1; y1), B=(x2; y2), C=(x3; y3), \text{ and } D=(x4; y4) \text{ are the intersections of } L:x+y=20 \text{ and the four curves } \Gamma 1:y=2^x, \Gamma 2:y=3^x, \Gamma 3:y=\log_2 x, \text{ and } \Gamma 4:y=\log_3 x, \text{ respectively.}$  Which of the following statements is true?

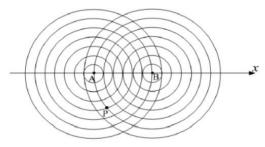
- (a)  $\overline{AB} < \overline{CD}$ .
- (b)  $\overline{AC} = 12$ .
- (c) y1 + y3 = 20.
- (d) x1 x2 = x3 x4.
- (e) 以上皆非. None of the above.
- 13. O 是複數平面上的原點, A = 1 + i,  $B = -1 + \sqrt{3}i$ ,  $C = A \times B$ ,  $A \times D = B$ . 下列敘述何者正確?

On complex plane, O is the origin, A = 1 + i,  $B = -1 + \sqrt{3}i$ ,  $C = A \times B$ ,  $A \times D = B$ . Which of the following statements is true?

- (a)  $\angle AOC = 105^{\circ}$ .
- (b)  $\overline{OC} = 4$ .
- (c)  $\overline{OA} = \overline{OD}$ .
- (d)  $\angle AOD = 45^{\circ} \circ$ .
- (e) 以上皆非. None of the above.
- 14.  $0^\circ < \theta < 90^\circ$ ,且 $\sin \theta \cdot \cos \theta \cdot \frac{1}{\cos \theta}$  成等差數列,則 $\tan \theta = ?$  If  $0^\circ < \theta < 90^\circ$ , and  $\sin \theta \cdot \cos \theta \cdot \frac{1}{\cos \theta}$  form an arithmetic progression, what is  $\tan \theta$ ?
  - (a)  $\frac{-1+\sqrt{5}}{2}$ .
  - (b)  $\frac{1+\sqrt{5}}{2}$ .
  - (c)  $-1+\sqrt{5}$
  - (d)  $1+\sqrt{5}$ .
  - (e) 以上皆非. None of the above.

15. 在直角坐標平面上, 分別以A(-3,0), B(3,0) 為圓心, 依1 單位、2 單位、…、7 單位做一系列的同心圓。若P=(a,b)的位置如圖, 為其中兩個圓的交點, 則下列哪些選項是正確的?

For the following figure, A = (-3, 0), B = (3, 0). The radii of the concentric circles centered at A or B are 1,2, 3, ..., 7. P = (a, b) is the intersection of two circles (shown in the figure). Which of the following statements is true?

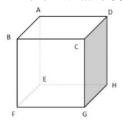


- (a) △ABP 是直角三角形. △ABP is right triangle.
- (b)  $\frac{a^2}{25} + \frac{b^2}{16} = 1$ .
- (c)  $a^2 \frac{b^2}{9} = 1$ .
- (d)  $a^2 + (b-3)^2 = 36$ .
- (e) 以上皆非. None of the above.
- 16. 若  $\vec{a}$  ,  $\vec{b}$  ,  $\vec{c}$  , 是平面上的三個非零向量, 則下列敘述何者正確?

Suppose  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  are non-zero vectors on plane. Which of the following statements is true?

- (a)  $\vec{a} \perp \vec{b}$ ,  $\vec{b} \perp \vec{c} \rightarrow \vec{a} \perp \vec{c}$
- (b)  $|\vec{a} + \vec{b}| = |\vec{a} \vec{b}| \rightarrow \vec{a} \perp \vec{b}$
- (c)  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \rightarrow \vec{b} = \vec{c}$
- (d)  $|\vec{a}|^2 + |\vec{b}|^2 = |\vec{c}|^2 \to \vec{a} \perp \vec{b}$
- (e) 以上皆非. None of the above.

For the cube: ABCD-EFGH, if  $\theta$  is the dihedral angle of planes *BDE* and *ABCD*, then what is  $\cos^2 \theta$ ?



- (a)  $\frac{1}{3}$ . (b)  $\frac{2}{3}$ . (c)  $\frac{3}{4}$ .
- (d)  $\frac{5}{1}$
- (e) 以上皆非. None of the above.
- 18. 三角形 $\overline{ABC}$ 中, $\overline{AB}$  = 3, $\overline{AC}$  = 4, $\overline{BC}$  = 5, $\overline{PA}$  + $\overline{PB}$  + $\overline{PC}$  = $\overrightarrow{0}$  .  $\overline{QA}$  =  $\overline{QB}$  =  $\overline{QC}$  . 下列敘述何者正確?

In triangle  $\overline{AB} = 3$ ,  $\overline{AC} = 4$ ,  $\overline{BC} = 5$ ,  $\overline{PA} + \overline{PB} + \overline{PC} = \overline{0}$ .  $\overline{QA} = \overline{QB} = \overline{QC}$ .  $\overline{QA} = \overline{QB} = \overline{QC}$ QC. Which of the following statements is true?

- (a)  $\overline{PQ} : \overline{PA} = 2 : 1$ .
- (b)  $\overrightarrow{BP} = \frac{1}{3}\overrightarrow{BA} + \frac{1}{3}\overrightarrow{BC}$ .
- (c)  $\overline{CP} : \overline{BP} = 1 : 1$ .
- (d)  $\overline{BP} = \frac{\sqrt{13}}{3}$ .
- (e) 以上皆非. None of the above.
- 19.  $a, b, c \in \mathbb{R}$ ,  $\lim_{x \to 1} \frac{a(x-1)^3 + x^2 bx + c}{x-1} = 2$ , 以下敘述何者正確?

Which of the following statements is true?

- (a) c b = 1.
- (b) b = 1.
- (c) c = 0.
- (d) a 可以是任意實數. a can be any real number.
- (e) 以上皆非. None of the above.

20. f(x) 是一個三次實係數多項式. 1 + i 為f'(x) = 0 的一個根, 且f(0) = f'(1) = 1 下列敘述何者正確?

f(x) is a cubic polynomial with real coefficients. 1 + i is a root for f'(x) = 0, and f(0) = f'(1) = 1. Which of the following statements is true?

- (a) f(x) 有三個實根.
- (b) f(x) 有一正根.
- (c) f(x) 的反曲點在 $(1,\frac{1}{2})$ .
- (d) 當x > 2 時f''(x) > 0
- (e) 以上皆非. None of the above.
- 21. 在所有體積為2π 的圓柱中,表面積最小的圓柱,其高與圓柱的半徑比值為何?

Among all the cylinders whose common volume is  $2\pi$ , what is the ratio of the height and the radius of the cylinder with the smallest surface area?

- (a) 2.
- (b) 1.
- (c)  $\pi$ .
- (d)  $\frac{1}{3}$ .
- (e) 以上皆非. None of the above.
- 22. A, B, C 是2×2矩陣, 下列敘述何者正確?

If A, B, and C are  $2 \times 2$  matrices, which of the following statements is true?

- (a) 如果A 有反矩陣, 那麼AB = BA. If A has inverse matrix, then AB = BA.
- (b) AB = AC => B = C.
- (c) 如果A 有反矩陣, 且 $AB = CA => \det B = \det C$ . If A has inverse matrix and AB = CA, then  $\det B = \det C$ .
- (d)  $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$
- (e) 以上皆非. None of the above.
- 23. 設 $a_1 = 0$ ,  $b_1 = 1$ . 且 $\forall_n \in \mathbb{N}$ ,  $a_{n+1} = 0.5 \cdot a_n + 0.75 \cdot b_n$ ,  $b_{n+1} = 0.5 \cdot a_n + 0.25 \cdot b_n$ . 若 $a = \lim_{n \to \infty} a_n$  和 $b = \lim_{n \to \infty} b_n$  存在, 則(a, b)為何?

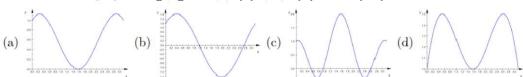
Suppose  $a_1 = 0$ ,  $b_1 = 1$ . and  $\forall_n \in \mathbb{N}$ ,  $a_{n+1} = 0.5 \cdot a_n + 0.75 \cdot b_n$ ,  $b_{n+1} = 0.5 \cdot a_n + 0.25 \cdot b_n$ . If  $a = \lim_{n \to \infty} a_n$  and  $b = \lim_{n \to \infty} b_n$  exist, what is (a, b) = ?

- (a) (0.75; 0.25)
- (b) (0.6, 0.4)
- (c) (0.5; 0.5)
- (d) (0.4; 0.6)
- (e) 以上皆非. None of the above.

- 24. 集合 $S(x) = \{a_n, n \in \mathbb{N} | a_1 = x, \forall k \in \mathbb{N}, a_{k+1} = 4a_k 1\}$ 下列敘述何者正確? For the set  $S(x) = \{a_n, n \in \mathbb{N} | a_1 = x, \forall k \in \mathbb{N}, a_{k+1} = 4a_k 1\}$ , which of the following statements is true?
  - (a) 如果p 在某個S(n) 中,  $n \in \mathbb{N}$ , 那麼p 除以4 會餘3. If  $p \in S(n)$ ,  $n \in \mathbb{N}$ , then the remainder of p divided by 4 is 3.
  - (b) 如果 $x \neq y$ ,  $S(x) \cap S(y)$  是空集合. If  $x \neq y$ ,  $S(x) \cap S(y)$  is an empty set.
  - (c) 考慮所有S(n) 中,  $n \in \mathbb{N}$ , 如果p 只在某一個S(n) 中, 那麼p 除以4 會餘1 或2. Among all S(n),  $n \in \mathbb{N}$ , if p is only contained in a specific set S(n), then the remainder of p divided by 4 is 1 or 2.
  - (e) 集合S(x),  $x \in \mathbb{R}$  的個數都會是無限多個. The number of elements in S(x) is always infinite if  $x \in \mathbb{R}$ .
  - (e) 以上皆非. None of the above.
- 25. 老師產生了兩個正整數a 跟b,1 < a < b < 10,並且告訴了Alice 兩個數的和,告訴了Bob 兩個數的積, Alice 跟Bob 都是很聰明的學生, 知道a 跟b 是正整數,也知道a 跟b 的範圍. 一開始Bob 說我不知道兩個數是什麼, Alice聽到後想了想也說我也還不知道兩個數是什麼. 然後Bob 聽到Alice 的說法就說他知道了, Alice 也說她知道了! 請問a, b 兩個數有可能是?

The teacher generated two integers a and b, 1 < a < b < 10. She told Alice a + b and told Bob  $a \cdot b$ . They tried to guess what a and b are. Alice and Bob are smart students and know the conditions of how the teacher generated the two numbers. First Bob said that he did not know the two integers. Alice heard and tried to solve. She told Bob that she did not know the two integers, either. Then Bob said that he knew the two integers now. Alice said that she also got the two integers. What could the two integers a and b be?

- (a) a = 3, b = 4.
- (b) a = 3, b = 6.
- (c) a = 2, b = 9.
- (d) a = 4, b = 6.
- (e) 以上皆非. None of the above.
- 26. 下列那一個圖形有可能是 $f(x) = \sin(x) + \cos(2x)$ ,  $0 \le x \le \pi$ ? Which of the following figures is  $f(x) = \sin(x) + \cos(2x)$ ,  $0 \le x \le \pi$ ?



(e) 以上皆非. None of the above.

27. X-Y 平面上四個點 $P_i=(x_i,\ y_i),\ P_1=(1,\ 2),\ P_2=(2,\ 0),\ P_2=(3.\ 1).\ P_4=(4,\ 8).\ 若直線<math>y=mx+b$  與這四個點的「鉛直的差距之平方和=  $\sum_{i=1}^4(y_i-b-mx_i)^2$  」最小。試求此直線為?

Four points  $P_i = (x_i, y_i)$ ,  $P_1 = (1, 2)$ ,  $P_2 = (2, 0)$ ,  $P_3 = (3, 1)$ ,  $P_4 = (4, 8)$ , are on the *X-Y* plane. If a line y = mx + b minimize the function  $\sum_{i=1}^{4} (y_i - b - mx_i)^2$ , what is the line?

- (a) y = -2.0 + 1.9x
- (b) y = -2.0 + 1.7x
- (c) y = -1.5 + 1.9x
- (d) y = -2.5 + 2.1x
- (e) 以上皆非. None of the above.
- 28. Alice 要買A, B, C 三種不同套件積木組出一艘大遊艇. 大遊艇需要A 套件40個, B 套件50個, C 套件45個. 但是商店中只有賣小房子(A 套件2個, B 套件4個, C 套件5個)及小車子(A 套件4個, B 套件3個, C 套件2個)。已知小房子一組賣200元, 小車子一組賣300元。請問Alice 要用最少的錢組出大遊艇需要小房子及小車子各買幾組?

Alice wants to make a big Lego yacht which consists of Lego suite A (40 units), B (50 units), and C (45 units). However, only small house (which consists of Lego suite A (2 units), B (4 units), and C (5 units)) and small car(which consists of Lego suite A (4 units), B (3 units), and C (2 units)) are sold in the store. The prices of a small house and a small car are 200 and 300, respectively. How many small houses and cars are needed if Alice want to minimize the cost?

- (a) 小房子10 組, 小車子4 組. Ten houses and four cars.
- (b) 小房子6 組, 小車子7 組. Six houses and seven cars.
- (c) 小房子8 組, 小車子6 組. Eight houses and six cars.
- (d) 小房子5 組, 小車子10 組. Five houses and ten cars.
- (e) 以上皆非. None of the above.
- 29.  $n \in \mathbb{N}, C(n, k) = \frac{n!}{k!(n-k)!}, \sum_{k=1}^{n} C(n, k) = ?$ 
  - (a) 1
  - (b) 2n 1
  - (c)  $2^n 1$
  - (d) C(n+1, 2)
  - (e) 以上皆非. None of the above.

30. P是一個平面, C是一個實心正方體. 下列敘述何者正確?

Support P is a plane and C is a solid cube. Which of the following statements is true?

(a) P跟C的交集不可能是正三角形.

The intersection of *P* and *C* cannot be a right triangle.

(b) *P* 跟 *C* 的交集不可能是五邊形.

The intersection of P and C cannot be a pentagon.

(c) P 跟C 的交集不可能是黄金矩形(兩邊長比值是 $\frac{\sqrt{5}+1}{2}$ ).

The intersection of P and C cannot be a golden rectangle.

(The ratio of two side lengths is  $\frac{\sqrt{5}+1}{2}$ ).

(d) P跟C的交集不可能是六邊形.

The intersection of *P* and *C* cannot be a hexagon.

- (e) 以上皆非. None of the above.
- 31. 觀察這些數字的規律:  $a_1 = 1$ ,  $a_2 = 11$ ,  $a_3 = 112$ ,  $a_4 = 1124$ ,  $a_5 = 11248$ ,  $a_6 = 1124$  $1124816, \cdots, a_n$ . 下一個數 $a_{n+1}$ 是將前一個數 $a_n$  的每個數字和接在 $a_n$ 的最後面. 請問a10 是幾位數?

Observe the pattern of sequence an below:  $a_1 = 1$ ,  $a_2 = 11$ ,  $a_3 = 112$ ,  $a_4 = 1124$ ,  $a_5 = 1124$ = 11248,  $a_6$  = 1124816, ...,  $a_n$ . The number  $a_{n+1}$  is generated by writing the sum of all digits in  $a_n$  after  $a_n$ . How many digits are there in  $a_{10}$ ?

- (a) 13.
- (b) 15.
- (c) 17.
- (d) 21.
- (e) 以上皆非. None of the above.
- 32. Bob 用同時丟5元及10元硬幣(不同大小)及它的正反面來猜這張考卷的答案. 首先,如果5元硬幣與10元硬幣為(正面,正面)他就選(a),(正面,反面)就選(b), (反面, 正面) 就選(c), (反面, 反面) 就選(d). 再來他還多投兩次檢查答案, 如果這 三次的結果都完全不一樣的話, 他就會選(e). 請問Bob 選到(e) 的機率有多大? Bob uses two different coins (big and small) to guess the answers of this exam. If the result of big and small coins is (head, head), (head,tail), (tail,head) and (tail,tail), he will choose (a), (b), (c), and (d), respectively. He checks twice after his first trial. If all the three results are totally different, he will choose (e). What is the probability that Bob will choose (e)?

  - (a)  $\frac{1}{4}$ . (b)  $\frac{3}{8}$ . (c)  $\frac{3}{16}$

- (d)  $\frac{9}{16}$ .
- (e) 以上皆非. None of the above.
- 33. 點Q 是點P(1, 2, 3) 關於2x + 2y + z = 0 的對稱點. 試求Q = ?

Q is the point of symmetry for P(1,2,3) with reference to 2x + 2y + z = 0. Find Q?

- (a) (-1, 0, 2).
- (b)  $(-2,-1,\frac{3}{2})$ .
- (c) (-3,-2, 1).
- (d) (-5,-4; 0).
- (e) 以上皆非. None of the above.
- 34. 下列的敘述(嫌疑人的自白)是跟答案(兇手)的選項配合起來,只有一個敘述 (人)是實話,其他四個敘述(人)都是謊話.請問正確答案(兇手)是哪一個? 注意,答案(兇手)的該行敘述不見得是實話.

If the answer(killer) is only one and only one of the following statements (from suspects) is true, the other four statements are lies, which one is the <u>CORRECT ANSWER</u> (killer)? Noted that the statement of the correct answer(killer) may be a lie.

- (a)「(a) 不是答案(兇手)」"(a) is not the answer(killer)."
- (b)「(c) 是答案(兇手)」"(c) is the answer(killer)."
- (c)「答案(兇手) 是(a) 跟(d) 其中一個」"The answer(killer) is one of (a) or (d)."
- (d)「(a) 是答案(兇手).」"(a) is the answer(killer)."
- (e)「(a),(b),(c),(d) 都不是答案(兇手)」"None of the above is the answer(killer)."
- 35. 下列敘述何者正確?

Which of the following statements is true?

- (a) 35 不會是兩個平方數相減. 35 is not the difference of two square numbers.
- (b) 第35 個質數小於100. The 35-th prime is smaller than 100.
- (c) 找不到35 的某個次方除以53 會餘1. We cannot find *p* such that the remainder of 35*p* divided by 53 is 1.
- (e) 小於35 有24 個正整數與35 互質. There are 24 positive integers which are smaller than 35 and relative prime to 35.
- (e) 以上皆非. None of the above.