

1. 假設 a, b 是整數, 且 $a^2 + b^2 = 596$ 。請問 $a + b$ 可以是多少?

Suppose that a and b are integers, and $a^2 + b^2 = 596$. What can $a + b$ be?

- (a) 40
- (b) 25
- (c) 12
- (d) 6
- (e) 以上皆非 None of the above

2. 假設 a, b 是正整數, 若 $2014 = 20a + 14b$, 以下敘述何者一定正確?

Suppose that a and b are integers and $2014 = 20a + 14b$. Which of the following statements is always true?

- (a) a 跟 b 共有14組解。 There are 14 pairs of solution (a, b) .
- (b) a 除以 7 的餘數是4。 When a is divided by 7, the remainder is 4 .
- (c) b 除以 5 的餘數是1。 When b is divided by 5, the remainder is 1.
- (d) a 跟 b 可以同為偶數。 Both a and b can be even.
- (e) 以上皆非 None of the above

3. 將 1, 2, 3, 4, 5 與兩個 + 號以及兩個 - 號, 以一個數字接一個運算符號再接一個數字做排列所形成的運算式中 (例: $5 + 4 - 3 - 2 + 1 = 5$), 算不出下列哪一個數?

Compute a number by re-arranging 1, 2, 3, 4, 5, two + signs and two - signs in a way that any two numbers or any two signs do not sit next to each other (e.g. $5 + 4 - 3 - 2 + 1 = 5$). Which of the following numbers cannot be the result of a possible rearrangement?

- (a) -1
- (b) 0
- (c) 1
- (d) 3
- (e) 以上皆非 None of the above

4. 令 α 與 β 是 $x^2 - 20x + 14$ 的兩個根且 $a_n = \alpha^n + \beta^n$, n 是正整數。若數列 $\langle a_n \rangle$ 滿足遞迴式 $a_{n+2} = pa_{n+1} + qa_n$, 試求 (p, q) ?

Let α and β be the two roots of $x^2 - 20x + 14$ and $a_n = \alpha^n + \beta^n$, where n is a positive integer. Suppose the sequence $\langle a_n \rangle$ satisfies the recursive formula $a_{n+2} = pa_{n+1} + qa_n$. What is (p, q) ?

- (a) (14, 20)
- (b) (20, 14)
- (c) (20, -14)
- (d) (14, -20)
- (e) 以上皆非 None of the above

5. 下列每個選項中的三點皆可以形成一個平面三角形, 請問哪個三角形關於直線 $3x + 4y = 25$ 的對稱圖形跟自己一樣?

In each of the following items, the three points form a triangle on the plane. Which of the following triangles has the following property: the symmetric figure of the triangle about the line $3x + 4y = 25$ is the same as itself?

- (a) $A(0, 0), B(7, 1), C(6, 8)$
 - (b) $A(0, 0), B(7, 1), D(3, 4)$
 - (c) $A(0, 0), B(7, 1), E(8, 6)$
 - (d) $D(3, 4), E(8, 6), F(0, 1)$
 - (e) 以上皆非 None of the above
6. 一個 iPad 是 15900 元, 若你手上有 100 元鈔票 200 張、500 元鈔票 40 張以及 1000 元鈔票 20 張, 請問共有幾種不需要找零錢的付錢方法?

Suppose the price of an iPad is 15900 and you have 200 100-dollar bills, 40 500-dollar bills, and 20 1000-dollar bills. How many ways can you pay for such an iPad exactly?

- (a) 68
- (b) 136
- (c) 272
- (d) 544
- (e) 以上皆非 None of the above

7. $5^{20.14}$ 的整數部分有幾位數字? ($\log_{10} 2 = 0.301$)

How many digits is the integer part of $5^{20.14}$? ($\log_{10} 2 = 0.301$)

- (a) 13
- (b) 14
- (c) 15
- (d) 16
- (e) 以上皆非 None of the above

8. 一個袋子中有六顆紅球、四顆白球和八顆藍球。如果隨機選出三顆球，至少一顆球是紅色的機率是多少？

In a bag there are 6 red, 4 white, and 8 blue balls. If 3 balls are drawn randomly from the bag, what is the probability that at least one ball is red?

- (a) $55/204$
- (b) $149/204$
- (c) $1/18$
- (d) $99/204$
- (e) 以上皆非 None of the above

9. 計算 $\cos(100^\circ) \cos(140^\circ) \cos(160^\circ)$ 。

Evaluate $\cos(100^\circ) \cos(140^\circ) \cos(160^\circ)$.

- (a) $-1/8$
- (b) $-3/8$
- (c) $-1/4$
- (d) $-\sqrt{3}/8$
- (e) 以上皆非 None of the above

10. 兩個數列 $\langle a_n \rangle, \langle b_n \rangle$ 滿足 $a_n = 2a_{n-1} + b_{n-1}$; $b_n = a_n - 2a_{n-1}$, 且 $a_0 = 2, a_1 = 3$ 。則 $c_n = a_n - a_{n-1}$ 是多少？

Given two sequences $\langle a_n \rangle, \langle b_n \rangle$ satisfying $a_n = 2a_{n-1} + b_{n-1}$ and $b_n = a_n - 2a_{n-1}$, with $a_0 = 2$, and $a_1 = 3$. What is $c_n = a_n - a_{n-1}$?

- (a) 2^{n-1}
- (b) 1
- (c) n
- (d) $(-1)^{n-1}$
- (e) 以上皆非 None of the above

11. 在 $\triangle ABC$ 中, $\overline{AB} = 9$, $\overline{AC} = 5$, $\cos \angle BAC = \frac{1}{6}$ 。設點 P, Q 分別在邊 $\overline{AB}, \overline{AC}$ 上, 並使得 $\triangle APQ$ 的面積是 $\triangle ABC$ 面積的 $\frac{1}{3}$, 則 \overline{PQ} 的最小可能值為?

In a triangle $\triangle ABC$, let $\overline{AB} = 9$, $\overline{AC} = 5$, $\cos \angle BAC = \frac{1}{6}$. Suppose P, Q are two points lying on $\overline{AB}, \overline{AC}$, respectively, such that the area of $\triangle APQ$ is one-third of that of $\triangle ABC$. What is the minimum length of \overline{PQ} ?

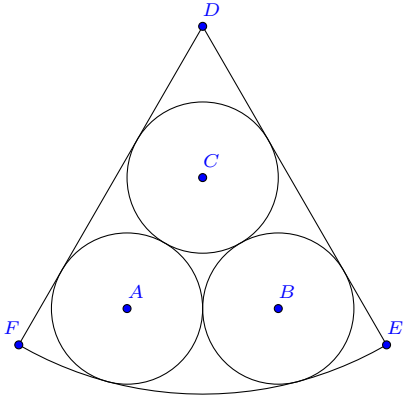
- (a) 5
 - (b) 4
 - (c) 3
 - (d) 2
 - (e) 以上皆非 None of the above
12. 在 $1, 2, 3, \dots, 2014$ 的自然數中, 有幾個數沒有 $2, 0, 1, 4$ 其中任何一個數字?
- How many positive integers among $1, 2, 3, \dots, 2014$ are not formed by any one of the following digits: $2, 0, 1$, and 4 ?
- (a) 128
 - (b) 256
 - (c) 512
 - (d) 474
 - (e) 以上皆非 None of the above

13. 設 z 是一個複數, $i = \sqrt{-1}$, 且 $|z - i| = 1$ 。求 $(3 + 4i)z$ 實部的最大值?

Let z be any complex number satisfying $|z - i| = 1$ where $i = \sqrt{-1}$. Find the maximum for the real part of $(3 + 4i)z$?

- (a) 2
- (b) 0
- (c) 1
- (d) 4
- (e) 以上皆非 None of the above

14. 下圖中圓 A, B, C 的半徑均相同, 互相外切, 並內切於扇形 DEF 。求扇形 DEF 與圓 A 的面積比值為?
 In the following figure, the three circles $A, B,$ and C have the same radius and are tangent to each other and also tangent to the sector DEF . Compute the ratio of the area of DEF to the area of the circle A ?



- (a) $\frac{1}{36}(1 + 4 \cos 15^\circ)^2$
 (b) $\frac{1}{36}(1 + 4 \sin 15^\circ)^2$
 (c) $\frac{1}{6}(1 + 4 \sin 15^\circ)^2$
 (d) $\frac{1}{6}(1 + 4 \cos 15^\circ)^2$
 (e) 以上皆非 None of the above
15. 計算 $\int \frac{x}{\sqrt{1-x^2}} dx$?
 Evaluate $\int \frac{x}{\sqrt{1-x^2}} dx$?
- (a) $-\sqrt{1-x^2} + C$
 (b) $\sqrt{1-x^2} + C$
 (c) $\frac{1}{2}\sqrt{1-x^2} + C$
 (d) $-\frac{1}{2}\sqrt{1-x^2} + C$
 (e) 以上皆非 None of the above

16. 將紅、藍、綠、白四種顏色塗在下面的四個格子中，一個格子塗一種顏色，而每個顏色可重複使用，但翻轉後相同視為同一種塗法 (即紅、藍、綠、白與白、綠、藍、紅視為同一種)。請問總共有幾種不同的塗法?

Four colors: red, blue, green, and white, are used to paint the following four squares. One square can be painted by just one color but different squares can be painted with the same color. Two kinds of painting are considered to be the same if one is the reflection of the other. For example, $(R,G,B,W) = (W,B,G,R)$. How many possible ways to paint the four squares?



- (a) 64
 (b) 128
 (c) 136
 (d) 163
 (e) 以上皆非 None of the above
17. 令 $a_k = \sum_{j=1}^k (-1)^{j-1}$; $b_n = \frac{1}{n} \sum_{k=1}^n a_k$. 試求極限值 $\lim_{n \rightarrow \infty} b_n = ?$
 Let $a_k = \sum_{j=1}^k (-1)^{j-1}$; $b_n = \frac{1}{n} \sum_{k=1}^n a_k$. Compute the limit $\lim_{n \rightarrow \infty} b_n = ?$
- (a) 1
 (b) 0
 (c) -1
 (d) $\frac{1}{2}$
 (e) 以上皆非 None of the above
18. 下列方程式有多少個實數解 x ?
 How many real numbers x are solutions to the equation $x = |x^2 - 2x| + |x^2 - 1|$?
- (a) 2
 (b) 0
 (c) 1
 (d) 4
 (e) 以上皆非 None of the above

19. 任選 a_1, a_2, \dots, a_8 八個不同的整數, 考慮兩兩之差所成的集合 $S = \{a_i - a_j\}$, 下列敘述何者正確?
 Given eight different integers a_1, a_2, \dots, a_8 . Let S be the set by collecting their differences, i.e., $S = \{a_i - a_j\}$. Which of the following statements is true?
- (a) S 中的最大值跟最小值的比值不會比 7 小. The ratio of the largest to the smallest value in S is not less than 7.
- (b) S 最多會有 28 個元素. There are at most 28 elements in S .
- (c) S 最少會有 28 個元素. There are at least 28 elements in S .
- (d) S 中至少有一個數字是 7 的倍數. At least one number in S is a multiple of 7.
- (e) 以上皆非 None of the above
20. 空間中三非零向量 $\vec{OA}, \vec{OB}, \vec{OC}$ 滿足 $\angle AOB = 30^\circ, \angle BOC = 45^\circ, \angle COA = 60^\circ$, 令 θ 為平面 AOB 及平面 BOC 的法向量夾角。試求 $|\cos \theta| = ?$
 Let $\vec{OA}, \vec{OB}, \vec{OC}$ be three non-zero vectors in three dimensional space. Suppose $\angle AOB = 30^\circ, \angle BOC = 45^\circ$, and $\angle COA = 60^\circ$. Let θ be the dihedral angle of plane AOB and BOC . Compute $|\cos \theta|$. (Note: the dihedral angle between two planes is the angle between their two unit normal vectors.)
- (a) $\sqrt{3} - \sqrt{2}$
- (b) $\sqrt{3}/2$
- (c) $\sqrt{2}/2$
- (d) $\frac{\sqrt{6}-\sqrt{2}}{4}$
- (e) 以上皆非 None of the above
21. 令 x, y 為小於 100 之正整數, 且 $x + y, x - y, xy, \frac{x}{y}$ 四數之和為 504, 求 $y = ?$
 Let x and y be two natural numbers both smaller than 100. The sum of $x + y, x - y, xy$ and $\frac{x}{y}$ is 504. What is y ?
- (a) 2
- (b) 3
- (c) 4
- (d) 5
- (e) 以上皆非 None of the above

22. 設矩陣 $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$, 其中 a, b, c 是實數且 $\det(A) = 2$ 。試求 $\det(A - A^{-1}) = ?$

Assume that the matrix $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ with a, b, c being real numbers and $\det(A) = 2$. What is $\det(A - A^{-1})$?

- (a) $\frac{9}{4}$
- (b) $\frac{9}{2}$
- (c) 3
- (d) 2
- (e) 以上皆非 None of the above

23. 設直線 $y = x + 5$ 與拋物線 $y^2 - 4y - 4x - 8 = 0$ 相交於 P, Q 兩點。令 F 為拋物線的焦點, 則 $\overline{PF} + \overline{QF} = ?$

Let P, Q be the intersections of the line $y = x + 5$ and the parabola $y^2 - 4y - 4x - 8 = 0$. Suppose the focus of the parabola is F . What is $\overline{PF} + \overline{QF} = ?$

- (a) 2
- (b) 4
- (c) 6
- (d) 8
- (e) 以上皆非 None of the above

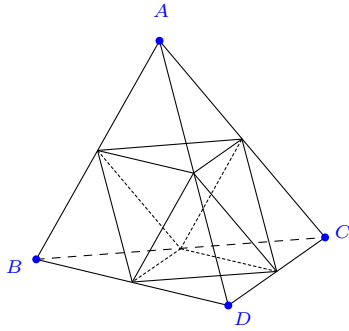
24. 請找出 $f(x) = \frac{2x^2+8}{x}$ 的最小值, 其中 $x > 0$ 。

Find the minimum of $f(x) = \frac{2x^2+8}{x}$ for $x > 0$.

- (a) 1.
- (b) 2.
- (c) 3.
- (d) 4.
- (e) 以上皆非 None of the above

25. 如圖所示，將一個正四面體的各邊中點用線段連接，可得到四個小正四面體及一個正八面體。如果原四面體的體積為12，那麼此正八面體的體積為？

Let $ABCD$ be a right tetrahedron with 6 edges of equal size. On the 6 edges there are 6 middle points. If we connect the middle points pairwise we can get four smaller right tetrahedrons and in the center it is a right octahedron. Suppose the volume of the original tetrahedron $ABCD$ is 12. Find the volume of the right octahedron?



- (a) 2
- (b) 4
- (c) 6
- (d) 8
- (e) 以上皆非 None of the above